Type Inference for Static Compilation of JavaScript

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Abstract
We present a type system and inference algorithm for a rich subset of JavaScript equipped with objects, structural subtyping, prototype inheritance, and first-class methods. The type system supports abstract and recursive objects, and is expressive enough to accommodate several standard benchmarks with only minor workarounds. The invariants enforced by the types enable an ahead-of-time compiler to carry out optimizations typically beyond the reach of static compilers for dynamic languages. Unlike previous inference techniques for prototype inheritance, our algorithm uses a combination of lower and upper bound propagation to infer types and discover type errors in all code, including uninvoked functions. The inference is expressed in a simple constraint language, designed to leverage off-the-shelf fixed point solvers. We prove soundness for both the type system and inference algorithm. An experimental evaluation showed that the inference is powerful, handling the aforementioned benchmarks with no manual type annotation, and that the inferred types enable effective static compilation.

Categories and Subject Descriptors D.3.3 [Programming Languages]: Language Constructs and Features; D.3.4 [Programming Languages]: Processors

Keywords object-oriented type systems, type inference, JavaScript

1. Introduction
JavaScript is one of the most popular programming languages currently in use [6]. It has become the de facto standard in web programming, and its growing use in large-scale, real-world applications—ranging from servers to embedded devices—has sparked significant interest in JavaScript-focused program analyses and type systems in both the academic research community and in industry.

In this paper, we report on a type inference algorithm for JavaScript developed as part of a larger, ongoing effort to enable type-based ahead-of-time compilation of JavaScript programs. Ahead-of-time compilation has the potential to enable lighter-weight execution, compared to runtimes that rely on just-in-time optimizations [5, 9], without compromising performance. This is particularly relevant for resource-constrained devices such as mobile phones where both performance and memory footprint are important. Types are key to doing effective optimizations in an ahead-of-time compiler.

JavaScript is famously dynamic; for example, it contains eval for runtime code generation and supports introspective behavior, features that are at odds with static compilation. Ahead-of-time compilation of unrestricted JavaScript is not our goal. Rather, our goal is to compile a subset that is rich enough for idiomatic use by JavaScript developers. Although JavaScript code that uses highly dynamic features does exist [39], data shows that the majority of application code does not require that flexibility. With growing interest in using JavaScript across a range of devices, including resource-constrained devices, it is important to examine the tradeoff between language flexibility and the cost of implementation.

The JavaScript compilation scenario imposes several desiderata for a type system. First, the types must be sound, so they can be relied upon for compiler transformations. Second, the types must impose enough restrictions to allow the compiler to generate code with good, predictable performance for core language constructs (Section 2.1 discusses some of these optimizations). At the same time, the system must be expressive enough to type check idiomatic coding patterns and make porting of mostly-type-safe JavaScript code easy. Finally, in keeping with the nature of the language, as well as to ease porting of existing code, we desire powerful type inference. To meet developer expectations, the inference must infer types and discover type errors in all code,
including uninvoked functions from libraries or code under development.

No existing work on JavaScript type systems and inference meets our needs entirely. Among the recently developed type systems, TypeScript [8] and Flow [3] both have rich type systems that focus on programmer productivity at the expense of soundness. TypeScript relies heavily on programmer annotations to be effective, and both treat inherited properties too imprecisely for efficient code generation. Defensive JavaScript [15] has a sound type system and type inference, and Safe TypeScript [36] extends TypeScript with a sound type system, but neither supports prototype inheritance. TAJS [29] is sound and handles prototype inheritance precisely, but it does not compute types for uninvoked functions; the classic work on type inference for SELF [12] has the same drawback. Choi et al. [21] present a JavaScript type system targeting ahead-of-time compilation that forms the basis of our type system, but their work does not have inference and instead relies on programmer annotations. See Section 6 for further discussion of related work.

This paper presents a type system and inference algorithm for a rich subset of JavaScript that achieves our goals. Our type system builds upon that of Choi et al. [21], adding support for abstract objects, first-class methods, and recursive objects, each of which we found crucial for handling real-world JavaScript idioms; we prove these extensions sound. The type system supports a number of additional features such as polymorphic arrays, operator overloading, and intersection types in manually-written interface descriptors for library code, which is important for building GUI applications.

Our type inference technique builds on existing literature (e.g., [12],[23],[35]) to handle a complex combination of language features, including structural subtyping, prototype inheritance, first-class methods, and recursive types; we are unaware of any single previous type inference technique that soundly handles these features in combination.

We formulate type inference as a constraint satisfaction problem over a language composed primarily of subtype constraints over standard row variables. Our formulation shows that various aspects of the type system, including source-level subtyping, prototype inheritance, and attaching methods to objects, can all be reduced to these simple subtype constraints. Our constraint solving algorithm first computes lower and upper bounds of type variables through a propagation phase (amenable to the use of efficient, off-the-shelf fixed-point solvers), followed by a straightforward error-checking and ascription phase. Our use of both lower and upper bounds enables type inference and error checking for uninvoked functions, unlike previous inference techniques supporting prototype inheritance [12],[29]. As shown in Section 2.3, sound inference for our type system is non-trivial, particularly for uninvoked functions; we prove that our inference algorithm is sound.

Leveraging inferred types, we have built a backend that compiles type-checked JavaScript programs to optimized native binaries for both PCs and mobile devices. We have compiled slightly-modified versions of six of the Octane benchmarks [1], which ranged from 230 to 1300 LOC, using our compiler. The modifications needed for the programs to type check were minor (see Section 5 for details).

Preliminary data suggests that for resource-constrained devices, trading off some language flexibility for static compilation is a compelling proposition. With ahead-of-time compilation (AOTC), the six Octane programs incurred a significantly smaller memory footprint compared to running the same JavaScript sources with a just-in-time optimizing engine. The execution performance is not as fast as JIT engines when the programs run for a large number of iterations, but is acceptable otherwise, and vastly better than a non-optimizing interpreter (details in Section 5).

We have also created six GUI-based applications for the Tizen [7] platform, reworking from existing web applications; these programs ranged between 250 to 1000 lines of code. In all cases, all types in the user-written JavaScript code were inferred, and no explicit annotations were required. We do require annotated signatures of library functions, and we have created these for many of the JavaScript standard libraries as well as for the Tizen platform API. Experiences with and limitations of our system are discussed in Section 5.

Contributions:

- We present a type system, significantly extending previous work [21], for typing common JavaScript inheritance patterns, and we prove the type system sound. Our system strikes a useful balance between allowing common coding patterns and enabling ahead-of-time compilation.
- We present an inference algorithm for our type system and prove it sound. To our best knowledge, this algorithm is the first to handle a combination of structural subtyping, prototype inheritance, abstract types, and recursive types, while also inferring types for uninvoked functions. This inference algorithm may be of independent interest, for example, as a basis for software productivity tools.
- We discuss our experiences with applying an ahead-of-time compiler based on our type inference to several existing benchmarks. We found that our inference could infer all the necessary types for these benchmarks automatically with only slight modifications. We also found that handling a complex combination of type system features was crucial for these programs. Experimental data points to the promise of ahead-of-time compilation for running JavaScript on resource-constrained devices.

2. Overview

Here we give an overview of our type system and inference. We illustrate some requirements and features of typing and type inference by way of a simple example. We also highlight
some challenges in inference, and show in more detail why previous techniques are insufficient for our needs.

2.1 Type System Requirements

Our type system prevents certain dynamic JavaScript behaviors that can compromise performance in an AOTC scenario. In many cases, such behaviors also reflect latent program bugs. Consider the example of Figure 1. We refer to the object literals as o1, o2, and o3. To keep our examples readable, we use a syntactic sugar for prototype inheritance: the expression `{a: 2} proto o1` makes o1 the prototype parent of the `{a: 2}` object, corresponding to the following JavaScript:

```javascript
function C() { this.a = 2 } // constructor
C.prototype = o1; new C()
```

In JavaScript, the `v2.m("foo")` invocation runs without error, setting v2.a to "foo". In SJS, we do not allow this operation, as v2.a was initialized to an integer value; such restrictions are standard with static typing.

JavaScript field accesses also present a challenge for AOTC. Figure 2 shows a runtime heap layout of the three objects allocated in Figure 1 (after line 6). In JavaScript, a field `x.f` first checks `x` for field `f`, and continues up `x`’s prototype chain until `f` is found. If `f` is not found, the read evaluates to undefined. Field writes `x.f = y` are peculiar. If `f` exists in `x`, it is updated in place. If not, `f` is created in `x`, even if `f` is available up the prototype chain. This peculiarity is often a source of bugs. For our example, the write to `this.a` within the invocation `v3.m(4)` on line 7 creates a new slot in o3 (dashed box in Figure 2), rather than updating o2.a.

Besides being a source of bugs, this field write behavior prevents a compiler from optimizing field lookups. If the set of fields in every object is fixed at the time of allocation—a fixed object layout [21]—then the compiler can use a constant indirection table for field offsets.[3] Fixed layout also establishes the availability of fields for reading / writing, obviating the need for runtime checks.[3]

In summary, our type system must enforce the following properties:

- **Type compatibility**, e.g., integer and string values cannot be assigned to the same variable.
- **Access safety** of object fields: fields that are neither available locally nor in the prototype chain cannot be read; and fields that are not locally available cannot be written.

These properties promote good programming practices and make code more amenable to compilation. Note that detection of errors that require flow-sensitive reasoning, like null dereferences, is out of scope for our type system: extant systems like TAJS [20] can be applied to find such issues.

2.2 The Type System

**Access safety.** In our type system, the fields in an object type `O` are maintained as two rows (maps from field names to types), `O^r` for _readable_ fields and `O^w` for _writeable_ fields. Readable fields are those present either locally or in the prototype chain, while writeable fields must be present locally (and hence must also be readable). Since o1 in Figure 1 only has local fields `d` and `m`, we have `O^r_1 = O^w_1 = (d, m)`.[2] For o2, the readable fields `O^r_2` include local fields `{a}` and fields `{d, m}` inherited from o1, so we have `O^r_2 = (d, m, a)` and `O^w_2 = {a}`. Similarly, `O^r_3 = (d, m, a, b)` and `O^w_3 = (b)`. The type system rejects writes to read-only fields; e.g., `v2.d = 2` would be rejected.

Detecting that the call `v3.m(4)` on line 7 violates access safety is less straightforward. To handle this case, the type system tracks two additional rows for certain object types: the fields that attached methods may read (`O^m`), and those that methods may write (`O^mw`). The typing rules ensure that such _method-accessed fields_ for an object type include the fields of the receiver types for all attached methods. Let `T^m_r` be the receiver type for method `m` (line 2). Based on the uses of `this` within `m`, we have `T^m_r = (d, a)` and `T^m_w = {a}` (again, writeable fields must be readable). Since `m` is the only method attached to `o1`, we have `O^m_{o1} = T^m_r = (d, a)` and `O^mw_{o1} = T^m_w = {a}`. Since `o2` and `o3` inherit `a` and have no other methods, we also have `O^m_{o2} = O^m_{o3} = (d, a)` and `O^mw_{o2} = O^mw_{o3} = {a}`.

With these types, we have `a ∈ O^mw_{o2}` and `a ∉ O^w_{o3}`; i.e., a method of `O_3` can write field `field a`, which is not locally present. Hence, the method call `v3.m(4) is unsafe.```
The type system considers $O_3$ to be abstract, and method invocations on abstract types are rejected. (Types for which method invocations are safe are concrete.) Similarly, $O_1$ is also abstract. Note that rejecting abstract types completely is too restrictive: JavaScript code often has prototype objects that are abstract, with methods referring to fields declared only in inheritors.

The idea of tracking method-accessed fields follows the type system of Choi et al. [20][21], but they did not distinguish between $mr$ and $mw$, essentially placing all access fields in $mw$. Their treatment would reject the safe call at line 4 whereas with $mr$, we are able to type it.

Subtyping. A type system for JavaScript must also support structural subtyping between object types to handle common idioms. But, a conflict arises between structural subtyping and tracking of method-accessed fields. Consider the following code:

```
1 p = cond()
2 ? { m: fun() { this.f = 1 }, f: 2 } // o1
3 : { m: fun() { this.g = 2 }, g: 3 } // o2
4 p.m();
```

Both $o1$ and $o2$ have concrete types, as they contain all fields accessed by their methods. Since $m$ is the only common field between $o1$ and $o2$, by structural subtyping, $m$ is the only field in the type of $p$. But what should the method-writeable fields of $p$ be? A sound approach of taking the union of such fields from $o1$ and $o2$ yields $\langle f, g \rangle$. But, this makes the type of $p$ abstract (neither $f$ nor $g$ is present in $p$), prohibiting the safe call of $p.m()$.

To address this issue, we adopt ideas from previous work [21][33] and distinguish prototypical types, suitable for prototype inheritance, from non-prototypical types, suitable for structural subtyping. Non-prototypical types elide method-accessed fields, thereby avoiding bad interactions with structural subtyping. For the example above, we can assign $p$ a non-prototypical concrete type, thereby allowing the $p.m()$ call. However, an expression $\{ \ldots \} \text{proto} p$ would be disallowed: without method-accessed field information for $p$, inheritance cannot be soundly handled. For further details, see Section 3.3.

2.3 Inference Challenges

As noted in Section 1, we found that no extant type inference technique was suitable for our needs. The closest techniques are those that reason about prototype inheritance precisely, like type inference for SELF [12] and the TAJS system for JavaScript [29]. Both of these systems work by tracking which values may flow to an operation (a “top-down” approach), and then ensuring the operation is legal for those values. They also gain significant scalability by only analyzing reachable code, as determined by the analysis itself.

But, this approach cannot infer types or find type errors in unreachable code, e.g., a function under development that is not yet invoked. Consider this example:

```
1 function f(x) {
2  var y = -x;
3  return x[1];
4 }
5 f(2);
```

Without the final call $f(2)$, the previous techniques would not find the (obvious) type error within $f$. This limitation is unacceptable, as developers expect a compiler to report errors in all code.

An alternative inference approach is to compute types based on how variables/expressions are used (a “bottom-up” approach), and then check any incoming values against these types. Such an approach is standard in unification-style inference algorithms, combined with introduction of parametric polymorphism to generalize types as appropriate [26]. Unfortunately, since our type system has subtyping, we cannot apply such unification-based techniques, nor can we easily infer parametric polymorphism.

Instead, our inference takes a hybrid approach, tracking value flow in lower bounds of type variables and uses in upper bounds. Both bounds are sets of types, and the final ascribed type must be a subtype of all upper bound types and a supertype of all lower bound types. Upper bounds enable type inference and error discovery for uninvoked functions, e.g., discovery of the error within $f$ above.

If upper bounds alone under-constrain a type, lower bounds provide a further constraint to inform ascription. For example, given the identity function $\text{id}(x) \{ \text{return} x; \}$, since no operations are performed on $x$, upper bounds give no information on its type. However, if there is an invocation $\text{id}(\text{"hi"})$, inference can use the lower bound information from "hi" to ascribe the type string $\rightarrow$ string. Note that as in other systems [3][8], we could combine our inference with checking of user-written polymorphic types, e.g., if the user provided a type $T \rightarrow T$ ($T$ is a type variable) for $\text{id}$.

Once all upper and lower bounds are computed, an assignment needs to be made to each type variable. A natural choice is the greatest lower bound of the upper bounds, with a further check that the result is a supertype of all lower bound types. However, if upper and lower bounds are based solely on value flow and uses, type variables can be partially constrained, with $\emptyset$ as the upper bound (if there are no uses) or the lower bound (if no values flow in). In the first case, since our type system does not include a top type, it is not clear what assignment to make. This is usually not a concern in unification-based analyses, which flow information across assignments symmetrically, but it is an issue in subtyping-based analyses such as ours.

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4 Throughout the paper, we call out extensions we made to enhance the power of Choi et al.’s type system.

5 We exclude $\top$ from the type system to detect more errors; see discussion in Section 4.2.
Particular care thus needs to be taken to soundly assign type variables whose upper bound is empty. A sound choice would be to simply fail in inference, but this would be too restrictive. We could compute an assignment based on the lower bound types, e.g., their least upper bound. But this scheme is unsound, as shown by the following example:

```javascript
function f(x) {
  var y = x; y = 2; return x.a+1;
}
```

Assume f is uninvoked. Using a graphical notation (edges reflect subtyping), the relevant constraints for this code are:

```
X' \rightarrow \langle a : \text{int} \rangle \quad Y' \leftarrow \text{int}
```

x has no incoming value flow, but it is used as an object with an integer a field (shown as the \(X' \rightarrow \langle a : \text{int} \rangle\) edge). For y, we see no uses, but the integer 2 flows into it (shown as the \(Y' \leftarrow \text{int}\) edge). A technique based solely on value flow and uses would compute the upper bound of \(X'\) as \(\{a : \text{int}\}\), the lower bound of \(Y'\) as \{int\}, and the lower bound of \(X'\) and upper bound of \(Y'\) as \(\emptyset\). But, ascribing types based on these bounds would be unsound: they do not capture the fact that if x is ascribed an object type, then y must also be an object, due to the assignment \(y = x\).

Instead, our inference strengthens lower bounds based on upper bounds, and vice-versa. For the above case, bound strengthening yields the following constraints (edges due to strengthening are dashed):

```
\downarrow_{\text{row}} \rightarrow X' \rightarrow \langle a : \text{int} \rangle \quad \downarrow_{\text{row}} \leftarrow Y' \leftarrow \text{int} \leftarrow \{\}
```

Given the type \(\langle a : \text{int} \rangle\) in the upper bound of \(X'\), we strengthen \(X'\)'s lower bound to \(\downarrow_{\text{row}}\) (a subtype of all rows), as we know that any type-correct value flowing into x must be an object. As \(Y'\) is now reachable from \(\downarrow_{\text{row}}\), \(\downarrow_{\text{row}}\) is added to \(Y'\)'s lower bound. With this bound, the algorithm strengthens \(Y'\)'s upper bound to \(\emptyset\), a supertype of all rows. Given these strengthened bounds, inference tries to ascribe an object type to y, and detects a type error with \(\text{int}\) in \(Y'\)'s lower bound, as desired. Apart from aiding in correctness, bound strengthening simplifies ascription, as any type variable can be ascribed the greatest-lower bound of its upper bound (details in Section 4.2).

3. Terms, Types, and Constraint Generation

This section details the terms and types for a core calculus based on that of Choi et al. [20], modelling a JavaScript fragment equipped with integers, objects, prototype inheritance, and methods. The type system includes structural subtyping, abstract types, and recursive types. As this paper focuses on constraint generation, rather than presenting the typing relation here, we show the constraint generation rules for inference instead, which also capture the requirements for terms to be well-typed. An associated technical report presents the full typing relation [18].

3.1 Terms

Figure 3 presents the syntax of terms. The metavariable \(a\) ranges over a finite set of fields \(\mathcal{A}\), which describe the fields of objects. Expressions \(e\) include base terms \(n\) (which we take to be integers), and variable declaration (let \(x = e_1\ in\ e_2\), use \((x)\), and assignment \((x := e)\). An object is either the empty object \(\{\}\) or a record of fields \(\{a_1 : e_1, \ldots, a_n : e_n\}\) proto \(e_p\), where \(e_p\) is the object’s prototype. We also have the null and the receiver, this.

Field projection \(e . a\) and assignment \(e_1 . a := e_2\) take the expected form. The calculus includes first-class methods (declared with the function syntax, as in JavaScript), which must be invoked with a receiver argument. Our implementation also handles first-class functions, but they present no additional complications for inference beyond methods, so we omit them here for simplicity. Additional details can be found in the extended version [18].

3.2 Types

Figure 4 presents the syntax of types. Types \(\tau\) include a base type (integers), objects (\(\nu\)), and two method types: unattached methods \(\{\tau_1 \Rightarrow \tau_2\}\), which retain the receiver type \(\tau_r\), and attached methods \(\{\} \Rightarrow \tau_2\), wherein the receiver type is elided and assumed to be the type of the object to which the method is attached. (If \(e_1 . a := e_2\) assigns a new method to \(e_1 . a\), \(e_2\) is typed as an unattached method. Choi et al. [21] restricted \(e_2\) to method literals, whereas our treatment is more general.)

Object types comprise a base type, \(\rho\), and a qualifier, \(q\). The base type is a pair of rows (finite maps from names to types), one for the readable fields \(r\) and one for the writeable
fields \(w\), \(r\), \(mr\), \(mw\). Well-formedness for object types (detailed in Section 3.3) requires that writeable fields are also readable.

We choose to repeat the fields of \(w\) into \(r\) in this way because it enables a simpler mathematical treatment based on row subtyping. Object types also contain recursive object types \(\mu a.\nu\), where \(\alpha\) is bound in \(\nu\) and may appear in field types.

Object qualifiers \(q\) describe the field accesses performed by the methods in the type, required for reasoning about access safety (see Section 2.2). A prototypal qualifier \(P(\mu r, \mu w)\) maintains the information explicitly with two rows, one for fields readable by methods of the type (\(mr\)), and another for method-writeable fields (\(mw\)). At a method call, the type system ensures that all method-readable fields are readable on the base object, and similarly for method-writeable fields. The NC and NA qualifiers are used to enable structural subtyping on object types, and are discussed further in Section 3.3.

### 3.3 Subtyping and Type Equivalence

Any realistic type system for JavaScript must support structural subtyping for object types. Figure 5 presents the subtyping rules for our type system. In the premises, we always write \(r_1 \equiv r_2\) as a shorthand for \(r_1 <: r_2 \land r_2 <: r_1\), and similarly \(r_1 \equiv r_2\) as a shorthand for \(r_1 <: r_2 \land r_2 <: r_1\).

The S-Row rule enables width subtyping on rows and row reordering, and the S-NONPROTO rule lifts those properties to nonprototypal object types (ignore the qualifier \(k\) for the moment). Note from S-Row that overlapping field types must be equivalent, disallowing depth subtyping—such subtyping is known to be unsound with mutable fields \(T\). Depth subtyping would be sound for read-only fields, but we disallow it to simplify inference.

As discussed in Section 2.2, there is no good way to preserve information about method-readable and method-writeable fields across use of structural subtyping. Hence, other than row reordering enabled by S-Row and S-PROTO, there is no subtyping between distinct prototypal types, which are the ones that carry method-readable and method-writeable information. To employ structural subtyping, a prototypal type must first be converted to a non-prototypal NC or NA type (distinction to be discussed shortly), using the S-PROTOCONC or S-PROTOABS rules. After this conversion, structural subtyping is possible using S-NONPROTO. Since non-prototypal types have no specific information about which fields are accessed by methods, they cannot be used for prototype inheritance or method updates; see Section 3.3.

The system also makes a distinction between concrete object types, on which method invocations are allowed, and abstract types, for which invocations are prohibited. For prototypal types, concreteness can be checked directly, by ensuring that all method-readable fields are readable on the object and similarly for method-writeable fields, i.e., \(r <: mr\)

\[\begin{align*}
\text{S-Row} & : \forall a \in \text{dom}(r). a \in \text{dom}(r) \land r[a] \equiv r'[a] \\
\text{S-NONPROTO} & : \forall r_1 <; r_2 \quad w_1 <; w_2 \quad k = \text{NC} \lor k = \text{NA} \\
\text{S-PROTO} & : \forall r_1 \equiv r_2 \quad w_1 \equiv w_2 \quad mr_1 \equiv mr_2 \quad mw_1 \equiv mw_2 \\
\text{S-PROTOCONC} & : \forall r <; mw \quad w <; mw \\
\text{S-PROTOABS} & : \forall r <; mw \quad \{r \mid w\}^{P(mr, mw)} <; \{r \mid w\}^{NC} \\
\text{S-CONCABS} & : \forall r <; w \quad \{r \mid w\}^{NC} <; \{r \mid w\}^{NA} \\
\text{S-METHOD} & : \tau_1 = \tau_2 <; | | \tau_1 = \tau_2 \\
\text{S-TRANS} & : \tau_1 <; \tau_2 <; \tau_3 = \tau_3 \\
\text{S-REFL} & : \tau <; \tau
\end{align*}\]

Figure 5: Subtyping and object-type well-formedness.

and \(w <; mw\) (the assumptions of the S-PROTOCONC rule). For non-prototypal types, we employ separate qualifiers NC and NA to distinguish concrete from abstract. Rule S-PROTOCONC only allows concrete prototypal types to be converted to an NC type, whereas rule S-PROTOABS allows any prototypal type to be converted to an NA type. The type system only allows a method call if the receiver type can be converted to the corresponding NA type, as this only removes the ability to invoke methods.

Revisiting the example in Figure 1 here are the types for objects \(O_1\), \(O_2\) and \(O_3\).

\[\begin{align*}
O_1 : \{\langle d : \text{int}, m : \text{[\_]} \Rightarrow \text{void} \rangle \mid \langle d, m \rangle\}^{P(d, a), (a)} \\
O_2 : \{\langle d : \text{int}, m : \text{[\_]} \Rightarrow \text{void}, a : \text{int} \rangle \mid \langle a \rangle\}^{P(d, a), (a)} \\
O_3 : \{\langle d : \text{int}, m : \text{[\_]} \Rightarrow \text{void}, a : \text{int}, b : \text{int} \rangle \mid \langle b \rangle\}^{P(d, a), (a)}
\end{align*}\]

(We omit writing the types of fields in rows duplicately.) In view of the subtyping relation presented above, the conversion of the prototypal type of \(O_2\) to a NC type is allowed.
While the type system of Choi et al. \[21\] restricts subtyping on prototypal types, their system does not have the notion of \textsc{NA}, and hence cannot type the example above.

\footnote{While the type system of Choi et al. \[21\] restricts subtyping on prototypal types, their system does not have the notion of \textsc{NA}, and hence cannot type the example above.}

As the receiver types for their methods would differ; this would make \texttt{p.m()} a type error. We exclude any other form of method subtyping, as we have not encountered a need for it in practice. More general function/method subtyping poses additional challenges for inference, due to contravariance, but extant techniques could be adopted to handle these issues \[22, 23\]; we plan to do so when a practical need arises.

Subtyping is reflexive and transitive. Recursive types are equi-recursive and admit \(\alpha\)-equivalence, that is:

\[
\mu\alpha.\nu <: \nu [\alpha \mapsto \mu\alpha.\nu]
\]

\[
\nu [\alpha \mapsto \mu\alpha.\nu] <: \mu\alpha.\nu
\]

\[
\mu\alpha.\nu <: \mu\beta. (\nu [\alpha \mapsto \beta])
\]

which directly implies:

\[
\mu\alpha.\nu \equiv \nu [\alpha \mapsto \mu\alpha.\nu]
\]

\[
\mu\alpha.\nu \equiv \mu\beta. (\nu [\alpha \mapsto \beta])
\]

Note that it is possible to expand a recursive type and then apply rule \textsc{S-NonProto} or \textsc{S-Proto} to achieve a form of width subtyping.

\textbf{Figure 5} also shows the well-formedness for types, \(\Delta \models \tau\), in the context \(\Delta\) representing a set of bound variables. All non-object non-variable types are well-formed (rule \textsc{WF-NonObject}). For object types, well-formedness requires that any writeable field is also readable, and that all field types are also well-formed (rules \textsc{WF-NC} and \textsc{WF-NA}). In prototypical types, well-formedness further requires that method-writeable fields are also method-readable, and that for any field \(a\) that is both readable and method-readable, the \(mr\) and \(r\) rows agree on \(a\)’s type (rule \textsc{WF-P}). Finally, rules \textsc{WF-REC} and \textsc{WF-VAR} respectively introduce and eliminate type variables to enable well-formedness of recursive types.

### 3.4 Constraint Language

Here, we present the constraint language used to express our type inference problem. Constraints primarily operate over families of row variables, rather than directly constraining more complex source-level object types. \textbf{Section 3.3} reduces inference for the source type system to this constraint language, and \textbf{Section 4} gives an algorithm for solving such constraints.

\textbf{Figure 7} defines the constraint language syntax. The language distinguishes type variables, which represent source-level types, and row variables, which represent the various components of a source-level object type. Each type variable \(X\) has five corresponding row variables: \(X^r\), \(X^w\), \(X^{mr}\), \(X^{sw}\), and \(X^{nl}\). The first four correspond directly to the \(r\), \(w\), \(mr\), and \(sw\) rows from an object type. To enforce the condition \(\models \{r \mid w\}^r\), \textbf{Figure 5} on all types, we impose the well-formedness conditions in \textbf{Figure 7} on all \(X\). The last
The strip variable, \( X, Y \) range over source types
variable sorts \( s := r \mid w \mid m \mid mw \mid all \)
row variables \( X^s, Y^s \) range over row / non-object types
literals \( L := \) int \( \downarrow_{\text{row}} \{ \ldots , a : X, \ldots \} \)
\[ |X_R| X_1 \Rightarrow X_2 \]
constraints \( C := L <: X^s \mid X^e <: L \mid X^e <: Y^s \mid C \wedge C \]
\[ | X^s <: Y^s \{ a_1, \ldots , a_n \} \mid \text{proto}(X) \mid \text{concrete}(X) \mid \text{strip}(X) \]
\[ \text{attach}(X_b, X_f, X_a) \]
acceptance criteria \( A := \text{notmethod}(X) \mid \text{notproto}(X) \)

well-formedness \( X^\text{all} <: X^e <: X^w \wedge X^\text{all} <: X^\text{mr} <: X^\text{mw} \)

Figure 7: Constraint language. We give the language syntax above the line, and well-formedness constraints below.

variable, \( X^\text{all} \), is used to ensure that types of fields in both \( r \) and \( mw \) are equivalent; if ascription fails for \( X^\text{all} \), there must be some inconsistency between \( X^r \) and \( X^\text{mr} \).

Type literals include int, unattached methods, and rows. The \( \downarrow_{\text{row}} \) type ensures a complete row subtyping lattice and is used in type propagation (see Section 4.1). To handle non-object types, row variables are “overloaded” and can be assigned non-object types as well. Our constraints ensure that if any row variable for \( X \) is assigned a non-row type \( \tau \), then all row variables for \( X \) will be assigned \( \tau \), and hence \( X \) should map to \( \tau \) in the final ascription.

The first three constraint types introduced in Figure 7 express subtyping over literals and row variables. We write \( X^s \equiv L \) as a shorthand for \( L <: X^s \wedge X^s <: L \) and \( X^s \equiv Y^t \) as a shorthand for \( X^s <: Y^t \wedge Y^t <: X^s \). Constraints can be composed together using the \( \wedge \) operator. A constraint \( X^s <: Y^s \setminus \{ a_1, \ldots , a_n \} \) means that \( X^s \) must be a subtype of the type obtained by removing the fields \( a_1, \ldots , a_n \) from \( Y^s \). Such constraints are needed for handling prototype inheritance, discussed further in Section 3.5.

The \( \text{proto}(X) \) and \( \text{concrete}(X) \) constraints enable inference of object type qualifiers. Constraint \( \text{proto}(X) \) means the ascribed type for \( X \) must be prototypal, while \( \text{concrete}(X) \) means the type for \( X \) must be a subtype of an NC type. The \( \text{strip}(X) \) constraint ensures \( X \) is assigned an attached method type, with no receiver type. Conversion of unattached method types to attached occurs during ascription (Section 4.2), so the constraint syntax only includes unattached method types.

The constraint \( \text{attach}(X_b, X_f, X_a) \) in Figure 7 handles method attachment to objects. For a field assignment \( e_1.a := e_2, X_b, X_f, \) and \( X_c \) respectively represent the type of \( e_1 \), the type of \( a \) in \( e_1 \)’s (object) type, and the type of \( e_2 \). Intuitively, this constraint ensures the following condition:

\[
(X_b^e <: X_R \mid \_ \Rightarrow \_) \implies \left( \text{proto}(X_b) \wedge X_b^{mr} <: X_R^t \wedge X_b^{mw} <: X_R^w \wedge \text{strip}(X_f) \right)
\]

That is, when \( X_a \) is an unattached method type with receiver \( X_R \), then \( X_b \) is prototypal, its method-readable and method-writeable fields must respectively include the readable and writeable fields of \( X_R \), and \( X_f \) is an attached method type. Note that \( \text{attach}(X_b, X_f, X_a) \) is not a macro for the above condition, as we do not directly support an implication operator in the constraint language. Instead, the condition is enforced directly during constraint propagation.

3.5 Constraint Generation

Constraint generation takes the form of a judgement

\[
X_R, \Gamma \vdash e : X \mid C,
\]

to be read as: in a context with receiver type \( X_R \) and inference environment \( \Gamma \), expression \( e \) has type \( X \) such that constraints in \( C \) are satisfied. Figure 8 presents rules for constraint generation; see Section 4.1 for an example.

Rules C-INT and C-VAR generate straightforward constraints. The constraints for C-ObjEMP ensure the empty object is assigned type \( \{ \_{\text{\cdot}} \}_{P(\text{\cdot})} \). The rule C-THIS is the only rule directly using the carrier’s type \( X_b \). The constraint \( X^w <=: \{ \_ \} \) in rule C-NULL ensures that \( X \) is assigned an object type. (Recall that \( X^s <: X^w \).)

The rule for variable declaration C-VARDECL passes on the constraints generated by its subexpressions \((C_1, C_2)\), with additional constraints \( Y^t <=: X^t \mid Y^w <=: X^w \), which are sufficient to ensure that the type \( Y_1 \) of the expression \( e_1 \) is a subtype of the fresh inference variable \( X_1 \) ascribed to \( x \) in the environment (no constraint on \( Y_1^{mr}, X_1^{mr}, Y_1^{mw} \) or \( X_1^{mw} \) is needed). Constraining both the \( r \) and \( w \) rows is consistent with the S-NonProto subtyping rule (Figure 5). We put \( x \) in the initialization scope of \( e_1 \) in order to allow for the definition of recursive functions.

The C-METHDECL rule constrains the type of the body \( e \) using fresh variables \( Y_1 \) and \( Y_2 \) for the parameter and receiver types. \( Y_R \) is constrained to be non-prototypal and concrete, as in any legal method invocation, the receiver type must be a subtype of an NC type. (Recall that prototypal types, if they are concrete, can be safely cast to NC.) The rule for method application C-METHAPPP ensures that the type \( X_1 \) of \( e_1 \) is concrete, and that its field \( a \) has a method type
\[ X_M \, \Gamma \vdash e : X \mid C \]

The `\text{strip}(X_M)` constraint ensures \( X_M \) is an `\text{attached}` method type. Note that a relation between \( X_1 \) and \( Y_R \) is ensured by an attach constraint when method \( a \) is attached to object \( e_1 \), following `\text{C-ATTRUPD}` or `\text{C-OBJLIT}`.

The last three rules deal more directly with objects. Constraint generation for attribute use `\text{C-ATTR}` applies to non-methods (for methods, `\text{C-METHAPP}` is used instead); the rule generates constraints requiring that \( e \) has an object type \( X_1 \) with a readable field \( a \), such that \( a \) does not have a method type (preventing detaching of methods). The attribute update rule `\text{C-ATTRUPD}` constrains \( a \) to be a `\text{writeable}` field of \( e_1 \) \((X^w_b <: \langle a : X_f \rangle)\), and ensures that \( X_f \) is a supertype of \( e_2 \)'s type \( X_v \). Finally, it uses the attach constraint to handle a possible method update.

Finally, the rule `\text{C-OBJLIT}` imposes constraints governing object literals with prototype inheritance. Its constraints dwarf those of other rules, as object literals encompass potential method attachment for each field (captured by attach \((X_1, X_v, Y)\)) in addition to prototype inheritance. For the literal type \( X \), the constraints ensure that the `\text{writeable}` fields \( X^\text{w} \) are precisely those declared in the literal. The readable fields must include those inherited from the prototype \((X' <: X'_p)\); note that \( X' <: X^\text{w} \) is imposed by well-formedness. Furthermore, the constraint \( X^\text{w}_p <: X^\text{w}_p \{ a_1, \ldots, a_n \} \) ensures that additional readable fields do not appear "out of thin air," by requiring that any

\[ O^2 \xrightarrow{a} V^w \xrightarrow{w} O^2 \]

\[ O^2 \xrightarrow{v} V_1 \xrightarrow{\text{attr}} O^2 \]

\[ O^2 \xrightarrow{a} V^\text{all} \xrightarrow{\text{all}} O^4 \]

\[ O^2 \xrightarrow{\text{attr}} V^\text{attr} \xrightarrow{\text{attr}} O^4 \]

\[ O^2 \xrightarrow{\text{attr}} V^\text{attr} \xrightarrow{\text{attr}} O^4 \]

\[ \cdots \xrightarrow{\text{attr}} Y_R \]

\[ \langle d : D \rangle \]

\[ \langle a : A \rangle \]

\[ \text{Figure 8: Constraint generation.} \]

\[ \text{Figure 9: Selected constraints for the example of Figure 1} \]

\[ \text{fields in } X' \text{ apart from the locally-present } a_1, \ldots, a_n \text{ be present in the prototype. Finally, we ensure both } X \text{ and } X_p \text{ are prototypal, and that any method-accessed fields from } X_p \text{ are also present in } X. \]

\[ \text{Example Figure 9 shows a graph representation of some constraints for } o_1, o_2, \text{ and } v \text{ from lines } 1, 3 \text{ of Figure 1. Nodes represent row variables and type literals, with variable names matching the corresponding program entities (} Y_R \text{ corresponds to } \text{this} \text{ on line } 2). \text{ Each edge } X^s \rightarrow Y^t \text{ represents a constraint } X^s <: Y^t. \text{ Black solid edges represent well-} \]
formedness constraints (Figure 7), while blue solid edges represent constraints generated from the code (Figure 8). Dashed or dotted edges are added during constraint solving, and will be discussed in Section 4.1.

We first discuss constraints for the body of the method declared on line 2. For the field read this.d, the C-ATTR rule generates $Y_R \vdash \langle d : D \rangle$. Similarly, the C-ATTRUPD rule generates $Y'_{Rw} \vdash \langle a : A \rangle$ for the write to this.a.

For the containing object literal o1, the C-OBLIT rule creates row variables for type $O_1$ and edges $O''_1 \leftrightarrow \langle d : \text{int} \rangle$ (due to type equality). It also generates a constraint $\text{attach}(O_1, M, F)$ (not shown in Figure 9) to handle method attachment to field $m$ ($F$ is the type of the line 2 function); we shall return to this constraint in Section 4.1. The assignment to v1 yields the V1 row variables and the $O''_1 \rightarrow V'_1$ and $O''_1 \rightarrow V''_1$ edges via C-VARDECL.

For e2 on line 3, C-OBLIT yields the $O''_2 \leftrightarrow \langle a : \text{int} \rangle$ edges for the declared a. The use of v1 as a prototype yields the constraints $O''_2 \rightarrow V'_1$, $O''_2 \rightarrow V'''_1$, and $O''_2 \rightarrow V'''_1$, capturing inheritance. We also have $V'_1 \backslash \langle a \rangle \rightarrow O''_2$ to prevent "out of thin air" readable fields on O2. Finally, we generate $\text{proto}(V_1)$ (not shown) to ensure V1 gets a prototypical type.

4. Constraint Solving

Constraint solving proceeds in two phases. First, type propagation computes lower and upper bounds for every row variable, extending techniques from previous work [22, 35]. Then, type ascription checks for type errors, and, if none are found, computes a satisfying assignment for the type variables.

4.1 Type Propagation

Type propagation computes a lower bound $\lfloor X^s \rfloor$ and upper bound $\lceil X^s \rceil$ for each row variable $X^s$ appearing in the constraints, with each bound represented as a set of types. Intuitively, $X^s$ must be ascribed a type between its lower and upper bound in the subtype lattice. Figure 10 shows the rules for type propagation. Given initial constraints $C$, propagation computes the smallest set of constraints $C'$, and the smallest sets of types $\lceil X^s \rceil$ and $\lfloor X^s \rfloor$ for each variable $X^s$, verifying the rules of Figure 10. In practice, propagation starts with $C' = C$ and $\lfloor X^s \rfloor = \lceil X^s \rceil = \emptyset$ for all $X^s$. It then iteratively grows $C'$ and the bounds to satisfy the rules of Figure 10 until all rules are satisfied, yielding a least fixed point.

Figure 10: Propagation rules.

The code uses JavaScript var syntax rather than let for the calculus.

(i) $C \subseteq C'$.
(ii) $X^{all} \ll X^i \ll X^w \ll C'$ and $X^{all} \ll X^{m} \ll C'$.

Subtyping

(iii) if $X^s : L \in C'$, then $L \in \lceil X^s \rceil$;
(iv) if $L \in X^s \in C'$, then $L \in \lceil X^s \rceil$;
(v) if $X^s : Y^s \in C'$, then $[X^s] \subseteq [Y^s]$ and $[Y^s] \subseteq [X^s]$;
(vi) if $X^s : Y^s \ll \{a_1, \ldots, a_n\} \in C'$, then for any $\langle F \\{a_1, \ldots, a_n\} \rangle$ to $\lceil X^s \rceil$;

Bound strengthening

(vii) if $L \in \lceil X^s \rceil$, then $\text{top}(L) \in \lceil X^s \rceil$;
(viii) if $L \in \lceil X^s \rceil$, then $\text{bot}(L) \in \lceil X^s \rceil$;

Prototypicality and concreteness

(ix) if $\text{proto}(Y) \in C'$, $X^s : Y^s \in C'$ and $X^w : Y^w \in C'$, then $\text{proto}(X) \in C'$, $X^s \ll Y^s \ll C'$ and $X^w \ll Y^w \ll C'$,

$x \text{mr} \in C'$,

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$x \text{mr} \in C'$,
\[ \bot(L) = \begin{cases} \bot_{\text{row}}, & \text{if } L \text{ is a row type} \\ L & \text{otherwise} \end{cases} \]

The rules ensure that any lower bound \([X^*]\) includes the best type information that can be inferred from \([X^*]\), and vice-versa.

Rules (ix)–(xi) handle the constraints for prototypicality and concreteness. Recall from Section 3.3 that a prototypical type is only related to itself by subtyping (modulo row reordering). So, if we have \(\text{proto}(Y)\) and \(X < Y\), it must be true that \(X \equiv Y\) and also \(\text{proto}(X)\) (to handle transitive subtyping). Rule (ix) captures this logic at the level of row variables. The subtyping rules (Figure 5) show that for any concrete (NC) type \(Y\), if \(X < Y\), then \(X\) must also be concrete, either as an NC type (S-NONPROTO) or a concrete prototypical type (S-PROTOCONC). Rule (x) captures this logic. Finally, if we have both \(\text{proto}(X)\) and \(\text{concrete}(X)\), Rule (xi) imposes the assumptions from the S-PROTOCONC rule of Figure 5 ensuring any method-accessed field is present in the type.

Rules (xii) and (xiii) handle method attachment. Rule (xii) enforces the meaning of attach as discussed in Section 3.4. To understand Rule (xiii) say that \(X\) and \(Y\) are both unattached method types such that \(X < Y\). If we add \(\text{strip}(Y)\) to make \(Y\) an attached method, \(X < Y\) still holds, by the S-METHOD subtyping rule (Figure 5). However, if \(\text{strip}(X)\) is introduced, then \(\text{strip}(Y)\) must also be added, or else \(X < Y\) will be violated.

Rules (xiv)–(xvi) introduce new type equalities that enable the inference to succeed in more cases (the rules are not needed for soundness). Rule (xiv) equates types of shared fields for any rows \(r_1 \in [X^*]\) and \(r_2 \in [X^*]\); the types must be equal since \(r_1 < r_2\) and the type system has no depth subtyping. Rule (xv) imposes similar equalities for two rows in the same upper bound, and Rule (xvi) does the same for methods.

**Example.** We describe type propagation for the example of Figure 9. For the graph, type propagation ensures that if there is a path from row variable \(X^*\) to type \(L\) in the graph, then \(L \in [X^*]\). E.g., given the path \(O_2^\text{s} \rightarrow O_2^w \rightarrow \langle a: \text{int} \rangle\), propagation ensures that \(\{a: \text{int}\} \subseteq \{O_2^w\}\). The new subtyping equality constraints added to \(C'\) in the rules in Figure 10 correspond to adding new edges to the graph. For the example, the C-METHDECL rule generates a constraint \(\mathcal{E}^r \equiv \{Y_1\} Y_1 \Rightarrow Y_2\) (not shown in Figure 9) for the method literal on line 2 of Figure 1. Once propagation adds \(\{Y_2\} Y_1 \Rightarrow Y_2\) to \(\mathcal{E}^r\), handling of the attach \((O_1, M, F)\) constraint (Rule (xii)) constrains the method-accessible fields of \(O_1\) to accommodate receiver \(Y_R\). Specifically, the solver adds the brown dashed edges \(O^w \rightarrow Y_1\) and \(O^w \rightarrow Y_R\).

The \(\text{proto}(V_1)\) constraint, combined with \(O_1^r < V_1^r\), leads the solver to equate all corresponding row variables for \(O_1\) and \(V_1\) (Rule (ix)). This leads to the addition of the red dotted edges in Figure 9. These new red edges make all the literals reachable from \(O_2^\text{all}\); e.g., we have path \(O_2^\text{s} \rightarrow O_2^w \rightarrow V_1^r \rightarrow O_1^f \rightarrow O_2^w \rightarrow \langle d: \text{int}, m: M\rangle\). So, propagation yields:

\[\{\langle a: \text{int}, d: \text{int}, m: M\rangle, \langle d: D\rangle, \langle a: A\rangle\} \subseteq \{O_2^\text{all}\}\]

Via Rule (xvi), the types of \(a\) and \(d\) are equated across the rows, yielding \(A = D \equiv \text{int}\). Hence, the inference discovers \texttt{this.a} and \texttt{this.d} on line 2 both have type \texttt{int}, without observing the invocations of \(m\).

**Implementation.** Our implementation computes type propagation using the iterative fixed-point solver available in WALA [10]. WALA’s solver accommodates generation of new constraints during the solving process, a requirement for our scenario. WALA’s solver includes a variety of optimizations, including sophisticated worklist ordering heuristics and machinery to only revisit constraints when needed. By leveraging this solver, these optimizations came for free and saved significant implementation work. As the sets of types and fields in a program are finite, the fixed-point computation terminates.

### 4.2 Type Ascription

**Algorithm 1** Type ascription.

```plaintext
1: procedure ASCRIBETYPE(X)
2:   if \(\text{strip}(X) \in C'\) then \text{strip} receivers in \([X^*], [X^*]\)
3:   for each \(X^*\) do
4:     if \(X^* = \emptyset\) then \(\Phi(X^*) \leftarrow \text{default}\)
5:     else
6:       \(\Phi(X^*) \leftarrow \text{glb}([X^*])\) \(\triangleright\) Fails if no glb
7:       for each \(L \in [X^*]\) do
8:         if \(L \not\in \Phi(X^*)\) then fail
9:       if \(\Phi(X^*) = \text{int} \lor \Phi(X^*) = \text{default}\) then
10:      \(\Phi(X) \leftarrow \Phi(X^*)\)
11:      else if \(\Phi(X^*)\) is method type then
12:       if notmethod\((X) \in C'\) then fail
13:      \(\Phi(X) \leftarrow \Phi(X^*)\)
14:      else \(\Phi(X^*)\) must be a row
15:       \(\rho \leftarrow \{\Phi(X') \mid \Phi(X^*)\}\)
16:       if \(\text{proto}(X) \in C'\) then
17:         if \(\text{notproto}(X) \in C'\) then fail
18:       \(\Phi(X) \leftarrow \rho^{\text{prot}(\Phi(X^*))}\)
19:       else if \(\text{concrete}(X) \in C'\) then \(\Phi(X) \leftarrow \rho^\text{NC}\)
20:      \(\Phi(X) \leftarrow \rho^\text{NA}\)
```

Algorithm 1 shows how to ascribe a type to variable \(X\), given bounds for all row variables \(X^*\) and the implied constraints \(C'\). Here, we assume each type variable can be ascribed independently, for simplicity; an associated technical report gives a slightly-modified ascription algorithm that handles variable dependencies and recursive types [18].

If required by a \(\text{strip}(X)\) constraint, line 2 handles stripping the receiver type in all method literals of \([X^*]\) and
For each $X^n$, we check if its upper bound is empty, and if so assign it the default type. For soundness, the same default type must be used everywhere in the final ascription; our implementation uses int. Conceptually, an empty set upper bound corresponds to a $\top$ (top) type. However we do not allow $\top$ in our system, as it would hide problems like objects and ints flowing into the same (unused) location, e.g., $x = \{\}; x = 3$.

If the upper bound is non-empty, we compute its greatest lower bound (glb) (line 9). The glb of a set of row types is a row containing the union of their fields, where each common field must have the same type in all rows. For example:

$$\text{glb}([\{a : \text{int}, b : \text{string}\}]) = \{a : \text{int}, b : \text{string}\}$$

If there is no glb for two upper bound types, ascription fails with a type error. Given a glb, the algorithm then checks that every type in the lower bound is a subtype of the glb (line 8). If this does not hold, then some use in the program may be invalid, including some value ascription fails (examples forthcoming).

Once all glb checks are complete, lines 9-20 compute a type for $X$ based on its row variables. If $\Phi(X')$ is an integer, method, or default type, then $X$ is assigned $\Phi(X')$. Otherwise, an object type for $X$ is computed based on its row variables. The appropriate qualifier is determined based on the presence of proto$(X)$ or concrete$(X)$ constraints in $C'$, as seen in lines 16-20. The algorithm also checks the acceptance criteria (Section 3.4), ensuring ascription failure if they apply (they are introduced by the C-METHDECL and C-ATTR rules in Figure 8).

Notice that NA is crucial to enable ascription based exclusively on glb of upper bounds. Absent NA, if an object of abstract type $\tau$ flows from $x$ to $y$, the types of $x$ and $y$ must be equal, as $\tau$ would have no supertypes in the lattice. Hence, qualifiers would have to be considered when deciding which fields should appear in object types, losing the clean separation in Algorithm 1. Note also, abstractness is not syntactic (in Figure 1, $v3$ is only abstract because of inheritance), so even computing abstractness could require another fixed point loop.

**Example.** Returning to $O2$ in the example of Figure 9, $[O2]\| = \{\{a : \text{int}\}, \{d : \text{int}, m : M\}\}$ after type propagation. Given type $[]\text{int} \Rightarrow \text{void}$ for $M$, $\text{glb}([O2]\|) = \{(a : \text{int}, d : \text{int}, m : \text{int} \Rightarrow \text{void})\}$. $\Phi(O2)$, $\Phi(O2\|)$, and $\Phi(O2\|\|)$ are computed similarly. Since we have $\text{proto}(O2)$ (by C-OBJLITT, Figure 8), at line 18 ascription assigns $O2$ the following type, shown previously in Section 3.3:

$$\{(d : \text{int}, m : \text{int} \Rightarrow \text{void}, a : \text{int}) | \langle a \rangle\}^{\text{proto}(d,a,\langle a \rangle)}$$

Using glb of upper bounds for ascription ensures a type captures what is needed from the term, rather than what is available. In Figure 1 note that $v3$ is only used to invoke method $m$. Hence, only $n$ will appear in the upper bound of $V_n\|$, and the type of $v3$ will only include $m$, despite the other fields available in object $v3$.

**Type error examples.** We now give two examples to illustrate detection of type errors. The expression $\langle\{a : 3\} \text{proto } \{\rangle . b \text{ erroneously reads a non-existent field } b$. For this code, the constraints are:

$$\langle \{a\} \rightarrow E' \longleftarrow O' \longleftarrow O^w \longleftarrow \{a : \text{int}\}$$

$E$ is the type of the empty object, and $O$ the type of the parenthesized object literal. The $\langle \{\rangle \rightarrow E'$ edges are generated by the C-OBJTEMP rule. As $O$ inherits from the empty object, we have $O' \rightarrow E'$, modeling inheritance of readable fields, and also $E' \rightarrow O'$, ensuring any readable field of $O$ except $a$ is inherited from $E$. Since $E$ is the empty object, these constraints ensure $a$ is the only readable field of $O$.

Propagation and ascription detect the error as follows. $\langle\{a : \text{int}\} \rightarrow \text{not added to } [E']$, though it is reachable, due to the $\{a\}$ filter on the edge from $E'$ to $O'$. Instead, we have $\{b : B\} \subseteq [E']$; intuitively, since $b$ is not present locally in $O$, it can only come from $E$. Further, we have $\{\langle\{\rangle\} \subseteq [E']$. Since $\langle\{\rangle \notin \{b : B\}$, line 8 of Algorithm 1 reports a failure.

As a second example, consider:

$$\langle\{m : \text{fun } () | \{\text{this.$\ast = 3\};\}_{3}\}_{m}\rangle$$

The invocation is in error, since the object literal $o$ is abstract (it has no $\ast$ field). Our constraints are:

$$\langle m : M \rightarrow O^w \longrightarrow O^mw \longrightarrow Y^w_R \longrightarrow \{f : \text{int}\}$$

As in Figure 9, the brown dashed edge stems from method attachment. From the invocation and C-METHAPP, we have concrete$(O)$. We also have proto$(O)$ (from C-OBJLITT), leading (via Rule (xi) to the dotted edge from $O^w$ to $O^mw$. Now, we have a path from $\langle m : M\rangle \rightarrow O^w$, and from $O^w$ to $\langle f : \text{int}\rangle$. Since $\langle m : M\rangle \notin \{f : \text{int}\}$, line 8 will again report an error.

### 4.3 Soundness of Type Inference

We prove soundness of type inference, including soundness of constraint generation, constraint propagation, and type ascription. We also prove our type system sound. Our typing judgment and proofs can be found in an associated technical report [13].

Our proof of soundness of type inference relies on three lemmas on constraint propagation and ascription, subtyping constraints, and well-formedness of ascribed types.

**Definition 1 (Constraint satisfaction).** We say that a typing substitution $\Phi$, which maps fields in $A$ to types in $T$, satisfies the constraint $C$ if, after substituting for inference variables in $C$ according to $\Phi$, the resulting constraint holds.
We experimented with a number of standard benchmarks (Table 1), among them a selection from the Octane suite \[1\] (the same ones used in recent papers on TypeScript \[36\] and ActionScript \[35\]), several from the SunSpider suite \[2\], and CDJS from Jetstream \[4\]. In all cases, our compiler relied on the inferred types to drive optimizations. A separate developer team also created six apps for the Tizen mobile OS (further details in Section 5.2). In all these programs, inference took between 1 and 10 seconds. We have used type inference on additional programs as well, which are not reported here; our regression suite runs over a hundred programs.

Table 1: Size is non-comment non-blank lines of code. Programs from the SunSpider suite appear on the left, those from Octane and Jetstream on the top right, and the Tizen apps on the bottom right.

<table>
<thead>
<tr>
<th>benchmark</th>
<th>size</th>
<th>benchmark</th>
<th>size</th>
</tr>
</thead>
<tbody>
<tr>
<td>access-binary-trees</td>
<td>41</td>
<td>splay</td>
<td>230</td>
</tr>
<tr>
<td>access-fannkuch</td>
<td>54</td>
<td>crypto</td>
<td>1296</td>
</tr>
<tr>
<td>access-nbody</td>
<td>145</td>
<td>Richards</td>
<td>290</td>
</tr>
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<td>access-nsieve</td>
<td>33</td>
<td>navier</td>
<td>355</td>
</tr>
<tr>
<td>bitops-3bit-bits-in-byte</td>
<td>19</td>
<td>deltablue</td>
<td>466</td>
</tr>
<tr>
<td>bitops-bits-in-byte</td>
<td>20</td>
<td>raytrace</td>
<td>672</td>
</tr>
<tr>
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<td>7</td>
<td>cdjs</td>
<td>684</td>
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<tr>
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<td>29</td>
<td>calc</td>
<td>979</td>
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<td>annex</td>
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<td>31</td>
<td>2048</td>
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<td>26</td>
<td>sensor</td>
<td>266</td>
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<tr>
<td>3d-cube</td>
<td>301</td>
<td></td>
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</tr>
</tbody>
</table>

5. Evaluation

All the features our type inference supports—structural subtyping, prototype inheritance, abstract types, recursive object types, etc.—were necessary in even this small sampling of programs. As one example, the raytrace program from Octane stores items of two different types in a single array; when read from the array, only an implicit “supertype” is assumed. Our inference successfully infers the common supertype. We also found the ability to infer types and find type errors in uninvoked functions to be useful in writing new code as well as typing legacy code.

5.1 Practical Considerations

Our implementation goes beyond the core calculus to support a number of features needed to handle real-world JavaScript programs. For user code, the primary additional features are support for constructors and prototype initialization (see discussion in Section 5.2) and support for polymorphic arrays and heterogeneous maps. The implementation also supports manually-written type declarations for external libraries: such declarations are used to give types for JavaScript’s built-in operators and standard libraries, and also for native platform bindings. These type declaration files can include more advanced types that are not inferred for user-written functions, specifically types with parametric polymorphism and intersection types. We now give further details regarding these extensions.

Maps and arrays  JavaScript supports dictionaries, which are key-value pairs where keys are strings (which can be constructed on the fly\[11\]) and values are of heterogeneous types. Our implementation supports maps, albeit with a homogeneous polymorphic signature \texttt{string} \rightarrow \tau, where \tau is any type. Our implementation permits array syntax (\texttt{a[i]}) for accessing maps, but not for record-style objects. Arrays are supported similarly, with the index type \texttt{int} instead of \texttt{string}. Note that maps (and arrays) containing different types can exist in the same program; we instantiate the \tau at each instance appropriately.

Constructors  Even though we present object creation as allocation of object literals, JavaScript programmers often use constructors. A constructor implicitly declares an object’s fields via assignments to fields of \texttt{this}. We handle constructors by distinguishing them syntactically (as functions with a capitalized name) and using syntactic analysis to discover which fields of \texttt{this} they write.

Operator overloading  JavaScript operators such as + are heavily overloaded. Our implementation includes a separate environment file with all permissible types for such operators; the type checker selects the appropriate one, and the backend emits the required conversion. Many of the standard functions are also overloaded in terms of the number or types of arguments and are handled similarly.

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10 For SunSpider, we chose all benchmarks that did not make use of Date and RegExp library routines, which we do not support. For Octane, we chose all benchmarks with less than 1000 LOC.

11 By contrast, object fields are fixed strings.
We support this pattern, provided that such field writes (including the write to the prototype field itself) appear immediately and contiguously after the constructor definition. Without this restriction, we cannot ensure in a flow-insensitive type system that the constructor is not invoked before all the prototype properties have been initialized. The code refactoring required to accommodate this restriction is straightforward (see Figure 11 for an example). We did not see any cases in which the prototype was updated more than once.

U (Unions). Lack of flow sensitivity also precludes type checking (and inference) for unions distinguished via a type test. This feature is useful in JavaScript programs, and we encountered it in one of the Octane programs. In the original crypto, the BigInteger constructor may accept a number, or a string and numeric base (arity overloading as well); we split the string case into a separate function, and updated call sites as appropriate. For cdjs, there were two places where the fields present in an object type could differ depending on the value of another field. We changed the code to always have all fields present, to respect fixed object layout.

P (Polymorphism). Although inferring polymorphic types is well understood in the context of languages like ML, its limits are less well understood in a language with mutable records and subtyping. We do not attempt to infer parametric polymorphism, although this feature is known to be useful in JavaScript programs and did come up in deltablue and cdjs. We plan to support generic types via manual annotations, as we already do for environment functions. For now, we worked around the issue with code duplication. See Figure 12 for an example.

I (Class-based Inheritance). Finally, JavaScript programs often use an ad hoc encoding of class-based inheritance: programmers develop their own shortcuts (or use libraries) that
use “monkey patching” and introspection. We cannot type these constructs, but our type system can support class-based inheritance via prototypal inheritance, with some additional verbosity (see Figure 13). The latest JavaScript specification includes class-based inheritance, which obviates the need for encoding classes by other means. We intend to support the new class construct in the future.

Usability by developers. With our inference system, developers remain mostly unaware of the types being inferred, as the inference is automatic and no explicit type ascription is generated. For inference failures, we invested significant effort to provide useful error messages that were understandable without knowledge of the underlying type theory. While some more complex concepts like intersection types are needed to express types for certain library routines, these types can be written by specialists, so developers solely interacting with the inference need not deal with such types directly.

More concretely, the Tizen apps listed in Table 1 were created by a team of developers who were not experts in type theory. The apps required porting of code from existing web applications (e.g., for tetris and 2048) as well as writing new UI code leveraging native Tizen APIs. To learn our subset of JavaScript, the developers primarily used a manual we wrote that described the restrictions of the subset without detailing the type inference system; an associated technical report gives more details on this manual [18].

5.3 More Problematic Constructs

Certain code patterns appearing in common JavaScript frameworks make heavy use of JavaScript’s dynamic typing and introspective features; such code is difficult or impossible to port to our typed subset. As an example, consider the json2.js program [14], a variant of which appears in Crockford [25]. A core computation in the program, shown in Figure 14, consists of a loop to traverse a JSON data structure and make in-place substitutions. In JavaScript, arrays are themselves objects, and like objects, their contents can be traversed with a for-in loop. Hence, the single loop at line 4 applies equally well to arrays and objects. Also note that in different invocations of walk, the variable v may be an array, object, or some value of primitive type.

Our JavaScript subset does not allow such code. We were able to write an equivalent routine in our subset only after significant refactoring to deal with maps and arrays separately, as shown in Figure 15. Moreover, we had to “box” values of different types into a common type to enable the recursive calls to type check. Clearly, this version loses the economy of expression of dynamically-typed JavaScript.

JavaScript code in frameworks (even non-web frameworks like underscore.js [15]) is often written in a highly introspective style, using constructs not supported in our subset. One common usage is extending an object’s properties in-place

11http://underscorejs.org/

15http://underscorejs.org/
```javascript
function walk(k, v) {
    var i, n;
    if (v && typeof v === object) {
        for (i in v) {
            n = walk(i, v[i]);
            if (n !== undefined) {
                v[i] = n;
            }
        }
    }
    return filter(k, v);
}

Figure 14: JSON structure traversal.
```

```javascript
function JSONVal() {
    this.tag = ...
    this.a = null; // array
    this.m = null; // map
    thisintval = 0; // int value
    this.strval = ""; // string value
}

function walk(k, v) { // v instance of JSONVal
    var i, j, n;
    switch (v.tag) {
        case Constants.INT:
        case Constants.STR:
            break;
        case Constants.MAP:
            for (var i in v.m) {
                n = walk(i, v.m[i]);
                if (n !== undefined) { v.m[i] = n; }
            }
            break;
        case Constants.ARRAY:
            for (j = 0; j < v.a.length; j++) {
                // j="" converts j to a string
                n = walk("", v.a[j]);
                if (n !== undefined) { v.a[j] = n; }
            }
            break;
        }
    return filter(k, v);
}

Figure 15: JSON structure traversal in our subset of JavaScript.
```

```javascript
Object.prototype.extend = function (dst, src) {
    for (var prop in src) {
        dst[prop] = src[prop];
    }
}

Figure 16: extend in JavaScript using the pattern shown in Figure 16. The code treats all objects—including those meant to be used as structs—as maps. Moreover, it also can add properties to dst that may not have been present previously, violating fixed-object layout. We do not support such routines in our subset.

As mentioned before, the full JavaScript language includes constructs such as eval that are fundamentally incompatible with ahead-of-time compilation. We also do not support adding or modifying behavior (aka “monkey patching”) of built-in library objects like Object.prototype (as is done in Figure 16). The community considers such usage as bad practices.

Even if we take away these highly dynamic features, there is a price to be paid for obtaining type information for JavaScript statically: either a programmer stays within a subset that admits automatic inference, as explored in this paper and requiring the workarounds of the kinds described in Section 5.2, or, the programmer writes strong enough type annotations (the last column of Table 2 shows the effort required in adding such annotations for the same Octane programs in [36]).

Whether this price is worth paying ultimately depends on the value one attaches to the benefits offered by ahead-of-time compilation.

5.4 The Promise of Ahead-of-Time Compilation

As mentioned earlier, we have implemented a compiler that draws upon the information computed by type inference (Section 2.1) and generates optimized code. The details of the compiler are outside the scope of the paper, but we present preliminary data to show that AOTC for JavaScript yields advantages for resource-constrained devices.

We measured the space consumed by the compiled program against the space consumed by the program running on v8, a modern just-in-time compiler for JavaScript. The comparative data is shown in Figure 17. The Octane programs were run with their default parameters. As the figure shows, ahead-of-time compilation yielded significant memory savings vs. just-in-time compilation.

We also timed these benchmarks for runtime performance on AOTC compiled binaries and the v8 engine. Figure 18 shows the results for one of the programs, deltablue; the figure also includes running time on duktape, a non-optimizing interpreter with a compact memory footprint. We observe that (i) the non-optimizing interpreter is quite a bit slower than the other engines, and (ii) for smaller numbers of iterations, AOTC performs competitively with v8. For larger iteration counts, v8 is significantly faster. Similar behavior was seen for all six Octane programs (see Figure 19). The AOTC slowdown over v8 for the largest number of runs ranged from 1.5X (navier) to 9.8X (raytrace). We expect significant further speedups from AOTC as we improve our

16Except for splay, which we ran for 80 as opposed to 8000 elements; memory consumption in splay is dominated by program data.
optimizations and our garbage collector. Full data for the six Octane programs, both for space and time, are presented in an associated technical report [18].

**Interoperability.** In a number of scenarios, it would be useful for compiled code from our JavaScript subset to interoperate with unrestricted JavaScript code. The most compelling case is to enable use of extant third-party libraries without having to port them, e.g., frameworks like jQuery[17] for the web or [19] for the many libraries available for Node.js [19]. Additionally, if a program contains dynamic code like that of Figure 14 or Figure 16 and that code is not performance-critical, it could be placed in an unrestricted JavaScript module rather than porting it.

Interoperability with unrestricted JavaScript entails a number of interesting tradeoffs. The simplest scheme would be to invoke unrestricted JavaScript from our subset (and vice versa) via a foreign function interface, with no shared heap. But, this would impose a high cost on such calls, due to marshalling of values, and could limit expressivity, e.g., passing functions would be difficult. Alternately, our JavaScript subset and unrestricted JavaScript could share the same heap, with additional type checks to ensure that inferred types are not violated by the unrestricted JavaScript. The type checks could “fail fast” at any violation, like in other work on gradual typing [39, 41, 44]. But, this could lead to application behavior differing on our runtime versus a standard JavaScript runtime, as the standard runtime would not perform the additional checks. Without “fail fast,” the compiled code may need to be deoptimized at runtime type violations, adding significant complexity and potentially slowing down code with no type errors. At this point, we have a work-in-progress implementation of interoperability with a shared heap and “fail fast” semantics, but a robust implementation and proper evaluation of these tradeoffs remain as future work.

6. Related Work

Related work spans type systems and inference for JavaScript and dynamic languages in general, as well as the type inference literature more broadly.

**Type systems and inference for JavaScript.** Choi et al. [20, 21] presented a typed subset of JavaScript for ahead-of-time compilation. Their work served as our starting point, and we built on it in two ways. First, our type system extends theirs with features that we found essential for real code, most crucially abstract types (see discussion throughout the paper). We also present a formalization and prove these extensions sound (see the technical report [18]). Second, whereas they relied on programmer annotations to obtain
types, we developed and implemented an automatic type inference algorithm.

Jensen et al. [29] present a type analysis for JavaScript based on abstract interpretation. They handle prototypical inheritance soundly. While their analysis could be adapted for compilation, it does not give a typing discipline. Moreover, their dataflow-based technique cannot handle partial programs, as discussed in Section 2.3.

TypeScript [8] extends JavaScript with type annotations, aiming to expose bugs and improve developer productivity. To minimize adoption costs, its type system is very expressive but deliberately unsound. Further, it requires type annotations at function boundaries, while we do global inference. Flow [3] is another recent type system for JavaScript, with an emphasis on effective flow-sensitive type inference. Although a detailed technical description is unavailable at the time of this writing, it appears that our inference technique has similarity to Flow’s in its use of upper- and lower-bound propagation [19]. Flow’s type language is similar to that of TypeScript, and it also sacrifices strict soundness in the interest of usability. It would be possible to create a sound gradually-typed version of Flow (i.e., one with dynamic type tests that may fail), but this would not enforce fixed object layout.

For TypeScript, a sound gradually-typed variant already exists [36], which we discuss shortly.

Early work on type inference for JavaScript by Thiemann [42] and Anderson et al. [14] ignored essential language features such as prototype inheritance, focusing instead on dynamic operations such as property addition. Guha et al. [28] present a core calculus $\lambda_\mathcal{S}$ for JavaScript, upon which a number of type systems have been based. TeJaS [30] is a framework for building type checkers over $\lambda_\mathcal{S}$ using bidirectional type checking to provide limited inference. Politz et al. [34] provide a type system enforcing access safety for a language with JavaScript-like dynamic property access.

Bhargavan et al. [15] develop a sound type system and inference for Defensive JavaScript (DJS), a JavaScript subset aimed at security embedding code in untrusted web pages. Unlike our work, DJS forbids prototype inheritance, and their type inference technique is not described in detail.

Gradual typing for JavaScript. Rastogi et al. [35] give a constraint-based formulation of type inference for ActionScript, a gradually-typed class-based dialect of JavaScript. While they use many related techniques—their work and ours are inspired by Pottier [22]—their gradually-typed setting leads to a very different constraint system. Their (sound) inference aims at proving runtime casts safe, so they need not validate upper bound constraints. They do not handle prototype inheritance, relying on ActionScript classes.

Rastogi et al. [36] present Safe TypeScript, a sound, gradual type system for TypeScript. After running TypeScript’s (unsound) type inference, they run their (sound) type checker and insert runtime checks to ensure type safety. Richards et al. [40] present StrongScript, another TypeScript extension with sound gradual typing. They allow the programmer to enforce soundness of some (but not all) type annotations using a specific type constructor, thus preserving some flexibility. They also use sound types to improve compilation and performance. Being based on TypeScript, both systems require type annotations, while we do not (except for signatures of external library functions). Moreover, they do not support general prototype inheritance or mutable methods, but rather rely on TypeScript’s classes and interfaces.

Type inference for other dynamic languages. Agesen et al. [12] present inference for Self, a key inspiration for JavaScript which includes prototype inheritance. Their constraint-based approach is inspired by Palsberg and Schwartzbach [32]. However, their notion of type is a set of values computed by data flow analysis, rather than syntactic typing discipline.

Foundations of type inference and constraint solving. Type inference has a long history, progressing from early work [26] through record calculus and row variables [45] through more modern presentations. Type systems for object calculi with object extension (e.g., prototype-based inheritance) and incomplete (abstract) objects extend back to the late 1990s [16, 17, 27, 37]. To our knowledge, our system is the first to describe inference for a language with both abstract objects and prototype inheritance.

Trifonov and Smith [43] describe constraint generation and solving in a core type system where (possibly recursive) types are generated by base types, $\perp, \top$, and $\rightarrow$ only. They introduce techniques for removing redundant constraints and optimizing constraint representation for faster type inference. Building on their work, Pottier [22, 23] crisply describes the essential ideas for subtyping constraint simplification and resolution in a similar core type system. We do not know of any previous generalization of this work that handles prototype inheritance. In both of these systems, lower and upper bounds for each type variable are already defined while resolving and simplifying constraints. Both lines of work support partial programs, producing schemas with arbitrary constraints rather than an established style of polymorphic type.

Pottier and Rémy [24] describe type inference for ML, including records, polymorphism, and references. Rémy and Vouillon [33] describe type inference for class-based objects in Objective ML. These approaches are based on row polymorphism rather than subtyping, and they do not handle prototype inheritance or non-explicit subtyping.

Aiken [13] gives an overview of program analysis in the general framework of set constraints, with applications to dataflow analysis and simple type inference. Most of our constraints would fit in his framework with little adaptation, and his resolution method also uses lower and upper bounds. His work is general and does not look into specific program construct details like objects, or a specific language like JavaScript.
Acknowledgements
We thank the anonymous reviewers for their detailed feedback, which significantly improved the presentation of the paper.

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